International Journal of Theoretical Physics, Vol. 29, No. 6, 1990

Special Relativistic Gravitational Theory

Zhang Junhao[1] **and Chen Xiang**[2]

Received September 15, 1989

Based on special relativity, we introduce a way to develop a new field theory from (1) the relativistic property of the particle coupling coefficient with the field, and (2) the field due to a static point source. As an example, we discuss a theory of electromagnetic and gravitational fields. The results of this special relativistic gravitational theory for the redshift and the deflection of light are the same as those deduced from general relativity. The results of experiments on the planetary perihelion procession shift and on an additional "short-range gravity" are more favorable to the special relativistic gravitational theory than to general relativity. We put forward a new idea to test experimentally whether the equivalence principle of general relativity is correct.

1. INTRODUCTION AND CONCLUSION

As is well known, the explanation of the anomalous perihelion shift of Mercury's orbit was a triumph of general relativity. However, between 1967 and 1974 there was considerable controversy over whether the perihelion shift was a confirmation or a refutation of general relativity. This controversy has not yet been concluded because the disagreement of the contribution of a solar quadrupole moment remains unresolved (Clifford, 1981). Some people hope to find a new gravitational theory that can avoid the possible difficulty of the perihelion shift.

Recently, Stacey *et al.* (1987*a*) deduced from their experiments some results which suggest that gravity contains a Yukawa potential contribution. One cannot deduce this additional Yukawa potential from the equation of the metric field of general relativity. However, it is impossible that one part of gravity is relative to a metric field while another part is independent of it. This fact means that we need a new gravitational theory different from general relativity.

[1,2]Department of Physics, Shantou University, Shantou, Guangdong, China.

So far, all interaction theories except gravitational theory are based on special relativity. Therefore, it is natural to explore the possiblity of developing a gravitational theory based on special relativity. This is the main purpose of this paper. Our discussion proves that the gravitational field must be a tensor of rank three, and that the gravitational force contains four terms from special relativity. But previous gravitational theories only take one term into account. So the consequences of a theoretical calculation of the redshift, light deflection, and the perihelion shift contradict the experimental results. This leads to the conclusion that one cannot explain the experimental results from a gravitational theory based on special relativity.

In the following discussion, we prove that if we take the other terms into account, then the results of the theoretical calculation of the redshift and light deflection are the same as those deduced from general relativity, and the result for the perihelion shift allows a slight difference from the corresponding result of general relativity. The important thing is that it is easy to find the field equation of an additional Yukawa potential. All the experimental results are more favorable to the special relativistic gravitational theory than to general relativity. Of course, we still need some new experiments to determine whether the special relativistic gravitational theory or general relativity is correct.

Generally the equivalence principle of general relativity has assumed that an accelerated frame in a region free of gravitational fields is equivalent to a rest frame in a given infinitesimal region. We can now consider the premises of this principle. First, the gravitational mass is equivalent to inertial mass. Second, the direction and the magnitude of gravity are independent of those of velocity.

From the point of view of the special relativistic gravitational field theory, some terms of gravity depend not only on mass, but also on velocity. Hence the principle of equivalence is incorrect in general. But it is difficult to observe these effects in experiment because the predicted effects of special relativity are very slight.

Discussing within the context of general relativity, Weinberg concluded that the principle of equivalence is tenable if the linear dimension of the particle is much smaller than the dimension of the gravitational field. On the other hand, we can easily see the difference from our discussion.

2. GENERAL CONSIDERATIONS

In general, the force exerted on a particle by a specific field depends upon the velocity of the particle. Suppose that the 4-dimensional force K_ν can be expanded in a series of powers of 4-dimensional velocity U_σ,

$$K_\nu(U_\sigma) = T_\nu + T_{\nu\sigma}U_\sigma + T_{\nu\sigma\rho}U_\sigma U_\rho + \cdots \tag{1}$$

where

$$T_\nu = K_\nu \Big|_{U_\sigma=0}, \qquad T_{\nu\sigma} = \frac{\partial K_\nu}{\partial U_\sigma}\Big|_{U_\sigma=0}, \qquad T_{\nu\sigma\rho} = \frac{1}{2}\frac{\partial^2 K_\nu}{\partial U_\sigma\,\partial U_\rho}\Big|_{U_\sigma=0}, \ldots \tag{2}$$

According to special relativity, both K_ν and U_σ are tensors of first rank. So T_ν, $T_{\nu\sigma}$, and $T_{\nu\sigma\rho}$ are tensors of first, second, and third rank, respectively. Substituting equation (1) into $K_4 = i\mathbf{K}\cdot\mathbf{u}/c$, where $\mathbf{u}$ is the 3-dimensional velocity and $\mathbf{K}$ is the 3-dimensional part of K_ν, we obtain

$$T_\nu U_\nu = 0, \qquad T_{\nu\sigma} U_\nu U_\sigma = 0, \qquad T_{\nu\sigma\rho} U_\nu U_\sigma U_\rho = 0 \tag{3}$$

Since every T is velocity independent, from equation (3) we obtain that the tensors defined in equation (2) must satisfy

$$\begin{aligned} &T_\nu = 0, \qquad T_{\nu\sigma} + T_{\sigma\nu} = 0 \\ &T_{\nu\sigma\rho} + T_{\sigma\rho\nu} + T_{\rho\nu\sigma} + T_{\nu\rho\sigma} + T_{\rho\sigma\nu} + T_{\sigma\nu\rho} = 0 \end{aligned} \tag{4}$$

Denote the nth term on the right-hand side of equation (1) by $K_\nu^{(n)}$, e.g.,

$$K_\nu^{(1)} = T_\nu, \qquad K_\nu^{(2)} = T_{\nu\sigma} U_\sigma, \qquad K_\nu^{(3)} = T_{\nu\sigma\rho} U_\sigma U_\rho \tag{5}$$

The force exerted on a particle by one type of field is proportional to the strength of the field. Using H_ν, $H_{\nu\sigma}$, and $H_{\nu\sigma\rho}$ to express the strength of field, then

$$T_\nu = \mu_0^{(1)} H_\nu, \qquad T_{\nu\sigma} = \mu_0^{(2)} H_{\nu\sigma}, \qquad T_{\nu\sigma\rho} = \mu_0^{(3)} H_{\nu\sigma\rho} \tag{6}$$

where H_ν, $H_{\nu\sigma}$, and $H_{\nu\sigma\rho}$ are tensors of first, second, and third rank, respectively. The coefficient $\mu_0^{(n)}$ is invariant under Lorentz transformation; it is defined as the particle coupling coefficient with the tensor field of rank n. The coupling coefficient with the electromagnetic field is the charge q, and the coupling coefficient with the gravitational field is the rest mass m_0.

Considering equation (5) and the relations

$$K_\nu = \gamma\{\mathbf{F}, i\mathbf{F}\cdot\mathbf{u}/c\}, \qquad U_\nu = \gamma\{\mathbf{u}, ic\} \tag{7}$$

where $\mathbf{F}$ and $\mathbf{u}$ are the 3-dimensional force and velocity, respectively, and $\gamma = (1 - u^2 c^{-2})^{-1/2}$, with c the velocity of light, we have the expression of the 3-dimensional force by the tensor field of rank n,

$$\begin{aligned} F_j^{(n)} = \mu^{(n)}[&(ic)^{n-1} H_{j444\cdots} + (ic)^{n-2}(H_{jk44\cdots} + H_{j4k4\cdots} + H_{j44k\cdots} + \cdots)u_k \\ &+ \cdots + H_{jklm\cdots} u_k u_l u_m \cdots] \end{aligned} \tag{8}$$

$$\mu^{(n)} = \mu_0^{(n)} \gamma^{n-2} \tag{9}$$

The construction of the force expression is $\mu^{(n)} * \{$polynomial of power of $u_k\}$, and its maximum power of u_k is $(n-1)$. The $\mu^{(n)}$ is called the coupling coefficient of the moving particle with the field of rank n. Correspondingly, $\mu_0^{(n)}$ is the coefficient of the rest particle. The relation between $\mu^{(n)}$ and γ can be called the relativistic property of the coupling coefficient. We should emphasize that the form of the 3-dimensional force exerted on a particle by a field as well as the dependence of the coupling coefficient of the moving particle on the velocity are completely determined by the rank of the tensor field. Conversely, we know that the relativistic property of the coupling coefficient can determine the rank of the tensor field.

Of course, another type of relation between K and U is

$$K_\nu^{(n)} = \mu_0 H_{\sigma\rho\cdots\omega} U_\nu U_\sigma \cdots U_\omega$$

but we may prove that $K_\nu^{(n)}$ satisfies

$$K_\nu^{(n)} = 0$$

for this type of relation, from $K_\nu^{(n)} U_\nu = 0$.

Now consider several simple situations.

(i) We assume that the coupling coefficient of the moving particle is $\mu^{(1)} = \mu_0^{(1)} \gamma^{-1}$, and the field acting on the particle is a first-rank tensor (4-dimensional vector field). According to equations (4) and (6), we have

$$H_\nu = 0 \tag{10}$$

This means that no such vector field exists. In other words, we cannot find a kind of field with coupling coefficient $\mu^{(1)}$. This agrees with present experimental observations.

(ii) Suppose that the coupling coefficient of the moving particle has nothing to do with the velocity of the particle. In this case the field must be a tensor of rank two: $\mu^{(2)} = \mu_0^{(2)}$.

(iii) The coupling coefficient of a moving particle with a tensor field of rank three is $\mu^{(3)} = \mu_0^{(3)} \gamma$. The gravitational mass of a particle is $m = m_0 \gamma$. So this implies that the gravitational field is a tensor of rank three.

There is reason to believe that a tensor field of rank higher than three may exist, but it has not yet been observed.

3. ELECTROMAGNETIC INTERACTION

3.1. The First Fundamental Hypothesis of Electromagnetic Theory

The most important experimental fact is that the charge of a particle is independent of its velocity, namely that the coupling coefficient of a moving particle is $\mu^{(2)} = q$. This experimental result can be taken as the first hypothesis of the electromagnetic theory. From this we immediately obtain the following result.

(i) The electromagnetic field is a tensor of rank two.

(ii) According to equations (4) and (6), the field strength must be an antisymmetric tensor,

$$H_{\nu\sigma} = -H_{\sigma\nu} \tag{11}$$

There are only six independent components. With the usual notation, we can write

$$H_{\nu\sigma} = \begin{pmatrix} 0 & B_3 & -B_2 & -iE_1/c \\ -B_3 & 0 & B_1 & -iE_2/c \\ B_2 & -B_1 & 0 & -iE_3/c \\ iE_1/c & iE_2/c & iE_3/c & 0 \end{pmatrix} \tag{12}$$

(iii) In this case, equation (8) has the simple form

$$F_i^{(2)} = \mu^{(2)}\{icH_{i4} + H_{ij}u_j\} \tag{13}$$

or

$$\mathbf{F}^{(2)} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{14}$$

This is just the Lorentz force, also only a consequence of the first hypothesis.

We need to take two points into consideration. Generally speaking, the 3-dimensional force is a function of 3-dimensional velocity. It can be expanded as

$$F_i(u_j) = F_i(u_j = 0) + \left(\frac{\partial F_i}{\partial u_j}\right)_{u_j=0} \cdot u_j + \frac{1}{2}\left(\frac{\partial^2 F_i}{\partial u_j\, \partial u_k}\right)_{u_j=0} \cdot u_j u_k + \cdots \tag{15}$$

Therefore we should know if it contains a term depending on $u_j u_k$. The reason that we do not observe such a term in the present investigation is perhaps due to the small particle velocity. Can we observe effects representing this term when $u \to c$? The answer derived from equation (13) is negative. It would lead to a collapse of special relativity if one could observe this term experimentally. So far such an observation has not been established.

In the general case, the second-rank tensor can be split into symmetric and antisymmetric parts,

$$H_{ij} = S_{ij} + \varepsilon_{ijk} B_k \tag{16}$$

where S_{ij} is the symmetric part of H_{ij}, and the direction of $\mathbf{u} \cdot \mathcal{S}$ is not perpendicular to $\mathbf{u}$. Equation (11) means that only the antisymmetric part appears in H_{ij}. Therefore, the direction of the velocity-dependent force is only perpendicular to $\mathbf{u}$. This agrees with current experiments.

3.2. The Second Fundamental Hypothesis of Electromagnetic Theory

The fields due to a static charge are

$$\mathbf{E} = q\mathbf{r}/(4\pi\varepsilon_0 r^3), \qquad \mathbf{B} = 0 \tag{17}$$

The electromagnetic field due to a moving charge can be derived from equation (17) by utilizing a Lorentz transformation. Thus, we can further establish the electromagnetic equations. We can adopt equation (17) as the second hypothesis of electromagnetic theory.

4. GRAVITATIONAL INTERACTION

4.1. The First Fundamental Hypothesis of Gravitational Theory

The first fundamental conclusion of gravitation experiments is the equivalence of inertial and gravitational mass; this means that the coupling coefficient of a particle with the gravitational field is $m = m_0\gamma = \mu^{(3)}$. In general, we call this conclusion of experiment the first fundamental hypothesis of gravitational theory. We can derive three conclusions from this fundamental hypothesis immediately.

(i) The gravitational field must be a tensor of rank three.

(ii) According to equations (4) and (6), the gravitational field strengths satisfy the relation

$$H_{\nu\sigma\rho} + H_{\sigma\rho\nu} + H_{\rho\nu\sigma} + H_{\nu\rho\sigma} + H_{\rho\sigma\nu} + H_{\sigma\nu\rho} = 0 \tag{18}$$

(iii) Considering equation (8), we obtain

$$F_i^{(3)} = \mu^{(3)}[-c^2 H_{i44} + ic(H_{ij4} + H_{i4j})u_j + H_{ijl}u_j u_l] \tag{19}$$

Here we use the symbol G_i to express the third term of this formula,

$$G_i = mH_{ijl}u_j u_l \tag{20}$$

From equation (18), we can prove that this term is the component of a vector which is perpendicular to $\mathbf{u}$; then it can be expressed in the form $mc^{-2}(\mathbf{u} \times \mathscr{R} \cdot \mathbf{u})$. Therefore, the gravitational force has the form

$$\mathbf{F}^{(3)} = m\left(\mathbf{E} + \frac{1}{c}\mathbf{u} \times \mathbf{B} + \frac{1}{c}\mathbf{u} \cdot \mathscr{P} + \frac{1}{c^2}\mathbf{u} \times \mathscr{R} \cdot \mathbf{u}\right) \tag{21}$$

where

$$\begin{aligned} E_i &= -c^2 H_{i44}, \qquad B_i = \tfrac{1}{2}ic^2(H_{jk4} - H_{kj4} + H_{j4k} - H_{k4j}) \\ P_{ij} &= \tfrac{1}{2}ic^2(H_{ij4} + H_{ji4} + H_{i4j} + H_{j4i}) \\ R_{il} &= \tfrac{1}{3}c^2(H_{jkl} - H_{kjl} + H_{jlk} - H_{klj}) \end{aligned} \tag{22}$$

where (i, j, k) takes an even permutation of $(1, 2, 3)$.

4.2. The Representation Subspace of the Lorentz Group

Equations (2) and (6) mean that $H_{\nu\sigma\rho}$ is symmetric for σ, ρ:

$$H_{\nu\sigma\rho} = H_{\nu\rho\sigma} \tag{23}$$

Then equation (18) becomes

$$H_{\nu\sigma\rho} + H_{\sigma\rho\nu} + H_{\rho\nu\sigma} = 0 \tag{24}$$

Equations (23) and (24) reduce the 64-dimensional representation space into a 20-dimensional subspace. This corresponds to the fact that we can only measure $3+3+6+8=20$ experimental values of field strengths [the coefficients of each term of equation (21)].

The third-rank tensor $H_{\nu\sigma\rho}$ can be confirmed by the direct product of a partial differential operator $\partial/\partial X_\nu$ with a tensor potential $A_{\sigma\rho}$ of rank two. Suppose that $A_{\sigma\rho}$ is a symmetric tensor,

$$A_{\sigma\rho} = A_{\rho\sigma} \tag{25}$$

In the general case, $H_{\nu\sigma\rho}$ can be expressed as

$$H_{\nu\sigma\rho} = a_1 \frac{\partial A_{\sigma\rho}}{\partial X_\nu} + a_2 \frac{\partial A_{\rho\nu}}{\partial X_\sigma} + a_3 \frac{\partial A_{\nu\sigma}}{\partial X_\rho} \tag{26}$$

From equations (23)-(25), we get

$$a_2 = a_3 = -\tfrac{1}{2} a_1 \tag{27}$$

If we choose $a_1 = 1$, then we have

$$H_{\nu\sigma\rho} = \frac{\partial A_{\sigma\rho}}{\partial X_\nu} - \frac{1}{2}\frac{\partial A_{\nu\rho}}{\partial X_\sigma} - \frac{1}{2}\frac{\partial A_{\nu\sigma}}{\partial X_\rho} \tag{28}$$

Substituting equation (28) into (22), we obtain

$$\begin{aligned}
E_i &= -c^2\left(\frac{\partial A_{44}}{\partial x_i} - \frac{\partial A_{i4}}{\partial x_4}\right) \\
B_i &= \frac{3ic^2}{2}\left(\frac{\partial A_{k4}}{\partial x_j} - \frac{\partial A_{j4}}{\partial x_k}\right) \\
P_{ij} &= \frac{ic^2}{2}\left(\frac{\partial A_{4j}}{\partial x_i} + \frac{\partial A_{4i}}{\partial x_j} - 2\frac{\partial A_{ij}}{\partial x_4}\right) \\
R_{il} &= c^2\left(\frac{\partial A_{kl}}{\partial x_j} - \frac{\partial A_{jl}}{\partial x_k}\right)
\end{aligned} \tag{29}$$

4.3. Discussion of the Gravitational Experimental Results in the Range $r > 1000$ m

4.3.1. *The Gravitational Field Due to a Static Spherical Body*

To further develop the equations of the gravitational field, we should discuss the possible form of the gravitational potential tensor from the angle of the space rotation group. Under the space rotation, A_{44} is scalar, A_{j4} and A_{4j} are 3-vectors, and A_{ij} is a 3×3 tensor. Because of the spherical symmetry of the source, they have the form

$$\begin{gathered} A_{44}=\xi(r), \qquad A_{j4}=A_{4j}=i\eta(r)x_j/r \\ A_{ij}=\zeta(r)\delta_{ij}+\theta(r)x_ix_j/r^2 \end{gathered} \tag{30}$$

Furthermore, all of the $A_{\sigma\rho}$ are time independent. Substituting (30) into (29), we obtain

$$\begin{aligned} E_i &= -c^2\frac{d\xi}{dr}\frac{x_i}{r} \\ B_i &= 0 \\ P_{ij} &= -c^2\left[\left(\frac{d\eta}{dr}-\frac{\eta}{r}\right)\frac{x_ix_j}{r^2}+\frac{\eta}{r}\delta_{ij}\right] \\ R_{il} &= c^2\frac{d\zeta_1}{dr}\frac{x_j\delta_{kl}-x_k\delta_{jl}}{r} \end{aligned} \tag{31}$$

where

$$\frac{d\zeta_1}{dr}=\frac{d\zeta}{dr}-\frac{\theta}{r}$$

When the source of the field is due to a static sphere, R_{il} is not dependent on ζ or θ singly but on ζ_1 only. Now let us assume that $\theta=0$ in the following discussion.

4.3.2. *Planetary Motion in Gravitational Fields Due to a Static Sphere*

Substituting (31) into (21), we obtain

$$\mathbf{F}^{(3)}=m\left\{\left[-c^2\frac{d\xi}{dr}-\left(\frac{d\eta}{dr}-\frac{\eta}{r}\right)cu_r+\frac{d\zeta_1}{dr}u^2\right]\mathbf{e}_r-\left(\frac{c\eta}{r}+\frac{d\zeta_1}{dr}u_r\right)\mathbf{u}\right\} \tag{32}$$

where $\mathbf{e}_r$ is the unit vector in the r direction, and u_r is the r component of $\mathbf{u}$, respectively. From this formula, we get

$$\frac{dw}{w}=\left(-\frac{d\xi}{dr}u_r-\frac{1}{c}\frac{d\eta}{dr}u_r^2-\frac{\eta}{cr}u_\varphi^2\right)dt \tag{33}$$

$$\frac{d(rmu_\varphi)}{rmu_\varphi}=-\left(\frac{c\eta}{r}+\frac{d\zeta_1}{dr}u_r\right)dt \tag{34}$$

where $w = mc^2$ and u_φ is the φ component of $\mathbf{u}$. The solution of (34) is

$$rmu_\varphi = (rmu_\varphi)|_{r_0} \exp\left[-c\int_{r_0}^{r} \frac{\eta}{r}\,dt - \zeta_1(r) + \zeta_1(r_0)\right] \tag{35}$$

If $\eta \neq 0$, then the angular momentum will decrease with time in the range $\eta > 0$ and increase in the range $\eta < 0$. But we never observe this phenomenon, so we say that the A_{j4} component of a gravitational field due to a static spherical body is zero, namely that

$$A_{j4} = A_{4j} = i\eta(r)x_j/r = 0 \tag{36}$$

Thus, we can derive two integrals of the motion of a planet,

$$w = w_0 \exp[-\xi(r) + \xi(r_0)] \tag{37}$$

and

$$rmu_\varphi = (rmu_\varphi)|_{r_0} \exp[-\zeta_1(r) + \zeta_1(r_0)] \tag{38}$$

Although this formula suggests that the angular momentum varies with r, in the following discussion we shall prove that $\zeta_1(r) \sim GM/c^2r$. Due to this, the deviation of the angular momentum of Mercury is within a few parts in 10^{14}, which can be considered in the error range of experiment. Therefore, this conclusion does not contradict the result of astronomical observation. From equations (37) and (38), we get

$$ru_\varphi = h \exp[\xi(r) - \zeta_1(r)] \tag{39}$$

$$u_r^2 + u_\varphi^2 = c^2 - k^2 \exp[2\xi(r)] \tag{40}$$

where h and k are constant. According to the inverse square law of gravity, $\xi(r)$ and $\zeta_1(r)$ can be expanded as

$$\xi(r) = \alpha_1 r^{-1} + \alpha_2 r^{-2} + \cdots \tag{41}$$

$$\zeta_1(r) = \beta_1 r^{-1} + \beta_2 r^{-2} + \cdots \tag{42}$$

where α_1, α_2, β_1, and β_2 are unknown coefficients which can be determined by present experimental results. From the conclusions of classical gravitational theory, we can forecast that $\alpha_2 \ll \alpha_1$ and $\beta_2 \ll \beta_1$. Substituting equations (41) and (42) into equations (39) and (40), we obtain the equation of orbital motion of a planet and get the perihelion shift of the planet,

$$\Delta\varphi = \frac{2\pi}{a(1-e^2)}\left(2\beta_1 - \alpha_1 + \frac{\alpha_2}{\alpha_1}\right) \tag{43}$$

where e and a are the eccentricity and semimajor axis of the orbit, respectively.

4.3.3. *Light Propagation in a Gravitational Field Due to a Static Spherical Body*

Since Maxwell's equations only describe an electromagnetic wave propagating in gravity-free space, we must develop a new set of equations which can describe electromagnetic wave propagation in gravitational fields. We discuss this problem in Section 5.

Here let us consider a photon as a particle with velocity c and mass $m = wc^{-2}$. Therefore, the photon is acted upon by gravity, and we can derive an equation for the orbit of a photon in a gravitational field. The following discussion shows that both methods give the same result.

According to the relations of momentum, energy, and mass of special relativity,

$$\mathbf{G} = m\mathbf{u}, \qquad w = mc^2 \tag{44}$$

we have

$$\mathbf{G}^2 - w^2c^{-2} = m^2(u^2 - c^2) = -m_0^2c^2 \tag{45}$$

for all kinds of particles, where m_0 is invariant under the Lorentz transformation. This relation agrees with the first hypothesis of gravitational theory. Therefore, for all kinds of particle (including the limiting case $u \to c$, $m_0 \to 0$), equations (21), (37), and (38) are still correct.

From equation (37), we have

$$w = w_0[1 - \xi(r) + \tfrac{1}{2}\xi^2(r) + \cdots] \tag{46}$$

where w_0 is the energy of a photon at $r = \infty$. According to the quantum theory of the electromagnetic field, $w = \hbar\nu$, we obtain

$$\nu - \nu_0 = -\nu_0[\alpha_1 r^{-1} + (\alpha_2 - \tfrac{1}{2}\alpha_1^2)r^{-2} + \cdots] \tag{47}$$

This is the formula of the redshift.

Now consider light deflected in the gravitational field of the Sun. Using equation (39) and

$$u_r^2 + u_\varphi^2 = c^2 \tag{48}$$

where u_r and u_φ are the r component and the φ component of the velocity of light, respectively, we derive the orbit equation of light

$$\left(\frac{d\rho}{d\varphi}\right)^2 = \frac{c^2}{h^2}\{1 + 2[\zeta_1(\rho) - \xi(\rho)] + \cdots\} - \rho^2 \tag{49}$$

where $\rho = r^{-1}$. Therefore the deflection angle of the light is

$$\delta = \frac{2(\beta_1 - \alpha_1)}{R} - \frac{2(\beta_1 - \alpha_1)^2}{R^2} \tag{50}$$

where R is the radius of the Sun.

4.3.4. Discussion of the Results of Gravitational Experiments

The experimental values of the redshift, the angle of deflection of light, and the planetary perihelion shift are

$$\nu - \nu_0 = \nu_0 \frac{GM}{c^2 R}$$

$$\delta = \frac{4GM}{c^2 R} \tag{51}$$

$$\Delta\varphi = \frac{6\pi GM(1+\Delta)}{a(1-e^2)}$$

respectively. Comparing the experimental values with the first-order approximate theoretical values, we have

$$\alpha_1 = -\beta_1 = \frac{-GM}{c^2}$$

$$\alpha_2 = -3\Delta \frac{G^2 M^2}{c^4} \tag{52}$$

$$\eta(r) = 0$$

Most physicists take $\Delta = 0$ here. From this point of view, three experimental values can be expressed by two parameters α_1 and β_1. It implies that special relativity can explain the internal relation of these three experiments. The disagreement over whether $\Delta = 0$ is exactly satisfied remains unresolved. If $\Delta \neq 0$, one has problems in general relativity. On the other hand, it is easy to give a description for the special relativistic gravitational theory.

The most important point of the result is that the preceding special relativistic gravitational theory only calculates the contribution of the E_i component of equation (21) (the contribution of α_1, α_2), and does not consider that the gravitational fields tensors of rank three. But this is not an essential error of special relativity.

4.4. The Second Fundamental Hypothesis of the Special Relativistic Gravitational Theory and the Equations of Gravitational Fields

Recently, Stacey *et al.* (1987*a,b*) discussed a Yukawa potential term of gravity. Correspondingly, we put forward a second hypothesis of gravitational theory from the experimental conclusion for $r > 1000$ m. Then we develop an auxiliary hypothesis from the experimental results for $r < 1000$ m.

4.4.1. The Second Hypothesis of Gravitational Theory Based on the Experimental Data for $r > 1000$ m

According to the experimental data for $r > 1000$ m, the second hypothesis of special relativistic gravitational theory is that the gravitational tensor potential due to a static spherical body is

$$\begin{aligned} A_{44} &= \frac{-GM}{c^2 r} - \frac{\lambda G^2 M^2}{2c^4 r^2} \\ A_{4i} &= A_{i4} = 0 \\ A_{ij} &= \left(\frac{GM}{c^2 r} + \frac{\beta_2}{r^2}\right)\delta_{ij} \end{aligned} \tag{53}$$

where $\lambda = 6\Delta$ and $\beta_2 \sim G^2M^2/c^4$. Here A_{44} is the solution of the nonlinear equation

$$\nabla^2 A_{44} + \lambda \frac{\partial A_{44}}{\partial x_j}\frac{\partial A_{44}}{\partial x_j} = \frac{4\pi G}{c^2}\rho_0 \tag{54}$$

in the case of a spherically symmetric source. Since β_2 does not appear in the first order of approximation, if we assume that $\beta_2 = \lambda G^2M^2/(2c^4)$, then it is acceptable for analyzing the experiment. Therefore, equation (53) is the solution of the equation

$$\nabla^2 A_{\nu\sigma} + \lambda \frac{\partial A_{\nu\omega}}{\partial x_j}\frac{\partial A_{\omega\sigma}}{\partial x_j} = \frac{4\pi G\rho_0}{c^2} N_{\nu\sigma} \tag{55}$$

where

$$N_{\nu\sigma} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{56}$$

Using the Lorentz transformation in equation (55), we get the equations of gravitational fields due to a moving source,

$$\frac{\partial^2 A_{\nu\sigma}}{\partial x_\xi\,\partial x_\xi} + \lambda \frac{\partial A_{\nu\omega}}{\partial x_\xi}\frac{\partial A_{\omega\sigma}}{\partial x_\xi} = \frac{-4\pi G\rho_0}{c^2}\left(\delta_{\nu\sigma} + \frac{2}{c^2}U_\nu U_\sigma\right) \tag{57}$$

or

$$\frac{\partial^2 A_{\nu\sigma}}{\partial x_j\,\partial x_\xi} + \lambda \frac{\partial A_{\nu\omega}\,\partial A_{\omega\sigma}}{\partial x_\xi\,\partial x_\xi} - \frac{\delta_{\nu\sigma}}{2}\left(\frac{\partial^2 A_{\alpha\alpha}}{\partial x_\xi\,\partial x_\xi} + \lambda \frac{\partial A_{\alpha\omega}\,\partial A_{\omega\alpha}}{\partial x_\xi\,\partial x_\xi}\right) = \frac{-8\pi G\rho_0 U_\nu U_\sigma}{c^4} \tag{58}$$

If the observation of planetary motion indicates that $\Delta = 0$, then the non-linear equation (58) reduces to the linear equation

$$\frac{\partial^2 A_{\nu\sigma}}{\partial x_\xi\,\partial x_\xi} - \frac{\delta_{\nu\sigma}}{2}\frac{\partial^2 A_{\alpha\alpha}}{\partial x_\xi\,\partial x_\xi} = \frac{-8\pi G\rho_0 U_\nu U_\sigma}{c^4} \tag{59}$$

4.4.2. The Revision of the Second Hypothesis in Terms of the Experimental Data for $r < 1000$ m

Stacey *et al.* suggest that we have reason to believe that the gravitational potential is

$$V = \frac{-GM}{r}(1 + \alpha e^{-r/\lambda}) \tag{60}$$

From this point of view, the second hypothesis of the gravitational theory can be revised: the potential tensor due to a sphere is

$$A_{\nu\sigma} = A_{\nu\sigma}^{(1)} + A_{\nu\sigma}^{(2)} \tag{61}$$

where $A_{\nu\sigma}^{(1)}$ has the form of equation (53), and

$$A_{44}^{(2)} = \frac{-\alpha GM}{c^2 r} e^{-r/\lambda} \tag{62}$$

This equation is the solution of

$$\nabla^2 A_{44}^{(2)} + \mu A_{44}^{(2)} = \frac{-8\pi\beta G\rho_0}{c^2} \tag{63}$$

in the case that the source is a sphere, where β can be determined from the experimental value α,

$$\beta = \tfrac{1}{3}\alpha\mu R^2 \exp(-\sqrt{-\mu}\, R) \tag{64}$$

The additional Yukawa potential in equation (60) is only relative to $A_{44}^{(2)}$. In order to establish the equations of the additional tensor potential field, it is necessary to obtain the other $A_{\nu\sigma}^{(2)}$ from experiments. However, the following equation is a possible form:

$$\frac{\partial^2 A_{\nu\sigma}^{(2)}}{\partial x_\xi\,\partial x_\xi} + \mu A_{\nu\sigma}^{(2)} = \frac{8\pi G\rho_0}{c^4}(\beta_1\delta_{\nu\sigma} + \beta_2 U_\nu U_\sigma) \tag{65}$$

where β_1, β_2 are constants determined by experiment.

This brief discussion has shown that even if the "short-range force" is further confirmed by experiment, this would not cause undue problems for the special relativistic gravitational theory.

5. THE EQUATIONS OF ELECTROMAGNETIC WAVES IN GRAVITATIONAL FIELDS AND THEIR SOLUTIONS

5.1. The Equations of Electromagnetic Wave in Gravitational Fields

We can prove that equation (21) has the form

$$\frac{\partial T_{\nu\sigma}}{\partial x_\sigma} = H_{\nu\omega\tau} T_{\omega\tau} \tag{66}$$

in the case of a continuous medium, where $T_{\nu\sigma}$ is the stress-energy-momentum tensor of the medium. Since the electromagnetic wave has its own energy, momentum, and mass, it is also acted upon by gravitational fields. As mentioned above, since equation (21) is correct whether $m_0 = 0$ or not, then equation (66) is also correct for electromagnetic waves. In this case, we can write

$$\frac{\partial}{\partial X_\sigma}(F_{\nu\alpha}F_{\alpha\sigma} - \tfrac{1}{4}\delta_{\nu\sigma}F_{\beta\alpha}F_{\alpha\beta}) = H_{\nu\omega\tau}(F_{\omega\alpha}F_{\alpha\tau} - \tfrac{1}{4}\delta_{\omega\tau}F_{\beta\alpha}F_{\alpha\beta}) \tag{67}$$

where $F_{\nu\alpha}$ is the electromagnetic field-strength tensor defined by (12) (the symbol $H_{\nu\alpha}$ is changed to $F_{\nu\alpha}$ here). Obviously, the equations of electromagnetic waves in gravitational fields have the form that the left-hand side of the equation is a partial differential of $F_{\nu\alpha}$, and the right-hand side of the equation is the product of the $H_{\nu\omega\tau}$ with $F_{\omega\tau}$. In the case of $H_{\nu\omega\tau} \to 0$, this set of equations tends to Maxwell's equations. So the left-hand sides of the equations of electromagnetic waves have a similar structure to Maxwell's equations. They contain two subsets, the vector $\partial F_{\nu\sigma}/\partial x_\sigma$ and the third-rank tensor $\partial F_{\mu\nu}/\partial x_\lambda + \partial F_{\nu\lambda}/\partial x_\mu + \partial F_{\lambda\mu}/\partial x_\nu$. At the same time, the right-hand sides of the equations must be a vector and a tensor of rank three correspondingly. The tensor of rank three is invariant under the cyclical permutation of (μ, ν, λ). Thus, the equations of electromagnetic waves have the form

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = a_1 H_{\alpha\beta\mu}F_{\beta\alpha} + a_2 H_{\beta\alpha\alpha}F_{\beta\mu} \tag{68}$$

$$\begin{aligned}\frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} + \frac{\partial F_{\lambda\mu}}{\partial x_\nu} &= b_1(H_{\mu\nu\alpha}F_{\alpha\lambda} + H_{\nu\lambda\alpha}F_{\alpha\mu} + H_{\lambda\mu\alpha}F_{\alpha\nu}) \\ &\quad + b_2(H_{\alpha\mu\nu}F_{\alpha\lambda} + H_{\alpha\nu\lambda}F_{\alpha\mu} + H_{\alpha\lambda\mu}F_{\alpha\nu}) \\ &\quad + b_3(H_{\mu\alpha\alpha}F_{\nu\lambda} + H_{\nu\alpha\alpha}F_{\lambda\mu} + H_{\lambda\alpha\alpha}F_{\mu\nu})\end{aligned} \tag{69}$$

We should choose two sets of suitable coefficients a_i and b_i in such a way that (67) could be derived from (68) and (69). For this purpose, we have

$a_2 = b_3 = 1/2$, $a_1 = -b_2 = 2/3$, and $b_1 = -4/3$. Using the notation in (12), we can rewrite (69) as

$$
\begin{aligned}
c^2\left(\nabla \times \mathbf{B} - c^{-2}\frac{\partial \mathbf{E}}{\partial t}\right) &= -\tfrac{1}{2}(\mathbf{B}\times\mathbf{E}^{(G)} + \mathbf{B}\cdot\mathcal{R} + \mathcal{R}\cdot\mathbf{B}) \\
&\quad + \frac{1}{c}[\mathbf{E}\cdot\mathcal{P} - \tfrac{1}{2}\mathbf{E}\, Sp(\mathcal{P}) - \tfrac{1}{3}\mathbf{E}\times\mathbf{B}^{(G)}] \\
c^2\left(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t}\right) &= -\tfrac{1}{2}(\mathbf{E}\times\mathbf{E}^{(G)} + \mathbf{E}\cdot\mathcal{R} + \mathcal{R}\cdot\mathbf{E}) \\
&\quad - \mathrm{c}[\mathbf{B}\cdot\mathcal{P} - \tfrac{1}{2}Sp(\mathcal{P})\mathbf{B} - \tfrac{1}{3}\mathbf{B}\times\mathbf{B}^{(G)}] \qquad (70) \\
c^2\nabla\cdot\mathbf{B} &= \tfrac{1}{2}\mathbf{B}\cdot(\mathbf{L}-\mathbf{E}^{(G)}) - \frac{2}{3c}\mathbf{E}\cdot\mathbf{B}^{(G)} \\
c^2\nabla\cdot\mathbf{E} &= \tfrac{1}{2}\mathbf{E}\cdot(\mathbf{L}-\mathbf{E}^{(G)}) + \frac{2c}{3}\mathbf{B}\cdot\mathbf{B}^{(G)}
\end{aligned}
$$

where E_i, B_i are the electromagnetic field strengths, $E_i^{(G)}$, $B_i^{(G)}$, L_i, P_{ij}, and R_{ij} are the gravitational field strengths, and

$$L_i = \varepsilon_{ijk}R_{jk}, \qquad Sp(\mathcal{P}) = p_{11} + p_{22} + p_{33} \qquad (71)$$

This set of equations is invariant under the following transformation:

$$E_i \to cB_i, \qquad cB_i \to -E_i \qquad (72)$$

5.2. The Propagation of Electromagnetic Waves in the Gravitational Fields Due to a Static Sphere

Using equations (31) and (36) and taking the center of the sphere as the origin of coordinates, we can rewrite (70) as

$$
\begin{aligned}
\frac{\partial \mathbf{E}}{\partial t} &= c^2\left(\nabla + \frac{1}{2}\frac{d\xi}{dr}\mathbf{e}_r\right)\times\mathbf{B} \\
-\frac{\partial \mathbf{B}}{\partial t} &= \left(\nabla + \frac{1}{2}\frac{d\xi}{dr}\mathbf{e}_r\right)\times\mathbf{E} \\
&\left[\nabla + \left(\frac{d\zeta_1}{dr} - \frac{1}{2}\frac{d\xi}{dr}\right)\mathbf{e}_r\right]\cdot\mathbf{E} = 0 \qquad (73) \\
&\left[\nabla + \left(\frac{d\zeta_1}{dr} - \frac{1}{2}\frac{d\xi}{dr}\right)\mathbf{e}_r\right]\cdot\mathbf{B} = 0
\end{aligned}
$$

If the electromagnetic wave is a cylindrical wave,

$$\frac{\partial E_i}{\partial z} = 0, \qquad \frac{\partial B_i}{\partial z} = 0 \qquad (74)$$

then, on the $z=0$ plane, (73) is split into two sets. One set is

$$\frac{\partial E_x}{\partial t}=c^2\left(\frac{\partial B_z}{\partial y}+\frac{1}{2}\frac{d\xi}{dr}\frac{yB_z}{r}\right)$$

$$\frac{\partial E_y}{\partial t}=-c^2\left(\frac{\partial B_z}{\partial x}+\frac{1}{2}\frac{d\xi}{dr}\frac{xB_z}{r}\right)$$

$$-\frac{\partial B_z}{\partial t}=\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}+\frac{1}{2}\frac{d\xi}{dr}\frac{xE_y-yE_x}{r}$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\left(\frac{d\zeta_1}{dr}-\frac{1}{2}\frac{d\xi}{dr}\right)\frac{xE_x+yE_y}{r}=0 \tag{75}$$

Another set can be obtained from the transformation

$$E_x\to cB_x,\qquad E_y\to cB_y,\qquad cB_z\to -E_z$$

In these two sets of equations, **B** is perpendicular to **E**.

Suppose that the phase of the electromagnetic wave is δ and the amplitudes of E_x, E_y and B_z are a_x, a_y, and b, respectively; then

$$E_x=a_x\exp(i\delta),\qquad E_y=a_y\exp(i\delta),\qquad B_z=b\exp(i\delta) \tag{76}$$

The situation differs from that of electromagnetic waves not in a gravitational field, in that all the amplitudes and k_x, k_y, ω are functions of r, where

$$k_x=\frac{\partial\delta}{\partial x},\qquad k_y=\frac{\partial\delta}{\partial y},\qquad \omega=\frac{\partial\delta}{\partial t} \tag{77}$$

Substituting (76) into (75), we obtain a set of complex equations, the imaginary part of which is

$$\begin{aligned}\omega a_x&=c^2bk_y, & -\omega a_y&=c^2bk_x\\ \omega b&=a_xk_y-a_yk_x, & k_xa_x+k_ya_y&=0\end{aligned} \tag{78}$$

The conclusions of (78) are as follows.

1. We have

$$a_x^2+a_y^2=c^2b^2,\qquad \text{or}\qquad |\mathbf{E}|=c|\mathbf{B}| \tag{79}$$

and in the general case, **B** is perpendicular to **E**.

2. $\mathbf{k}=(k_x, k_y)$, where **k** is perpendicular to **E** and **B**.
3. $\omega^2=c^2k^2$, where the phase velocity of light is c.

Moreover, the real part of this set of complex equations is

$$\frac{\partial b}{\partial y}=-\frac{1}{2}\frac{d\xi}{dr}\frac{yb}{r},\qquad \frac{\partial b}{\partial x}=-\frac{1}{2}\frac{d\xi}{dr}\frac{xb}{r} \tag{80}$$

$$\frac{\partial a_y}{\partial x}-\frac{\partial a_x}{\partial y}+\frac{1}{2}\frac{d\xi}{dr}\frac{xa_y-ya_x}{r}=0 \tag{81}$$

$$\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\left(\frac{d\zeta_1}{dr}-\frac{1}{2}\frac{d\xi}{dr}\right)\frac{xa_x+ya_y}{r}=0$$

if $\partial a_x/\partial t=\partial a_x/\partial t=\partial b/\partial t=0$. Therefore, we have

$$b=b_0\exp\{\tfrac{1}{2}[\xi(r_0)-\xi(r)]\} \tag{82}$$

Then the energy density of electromagnetic waves is

$$w=w_0\exp[\xi(r_0)-\xi(r)] \tag{83}$$

We consider that (83) is just (46), so we get the formula of the redshift immediately.

Now, we assume that at the point (r,φ) on the $0xy$ plane, the angle of k with the $0x$ axis is $\pi/2+\theta$; then we have

$$\frac{dr}{r\,d\varphi}=-\frac{\sin(\theta-\varphi)}{\cos(\theta-\varphi)} \tag{84}$$

and the equation

$$\frac{d\theta}{dr}=\left(\frac{d\zeta_1}{dr}-\frac{d\xi}{dr}\right)\frac{\cos(\theta-\varphi)}{\sin(\theta-\varphi)} \tag{85}$$

can be derived from (81). Thus, we get

$$\left(\frac{d\rho}{d\varphi}\right)^2=\frac{1}{K^2}\{1-2[\xi(\rho)-\zeta_1(\rho)]+\cdots\}-\rho^2 \tag{86}$$

where $\rho=r^{-1}$. Finally, due to equation (86), we obtain equation (50), namely, the formula of the deflection of light. Therefore, the consequence is that the redshift and the angle of light deflection derived from (70) and (73) are the same as those from (37) and (38).

6. EXPERIMENT CAN TEST THE EQUIVALENCE PRINCIPLE OF GENERAL RELATIVITY

As mentioned above, we cannot affirm which is correct, general relativity or the special relativistic gravitational theory, based on the experiments on the redshift, the deflection of light, and the planetary perihelion shift.

However, if $\Delta \neq 0$ is found in the observation of the perihelion shift, this will cause a problem for general relativity. On the other hand, if the existence of a "short-range force" is confirmed by experiment, then it will support the special relativistic gravitational theory. However, we hope to determine which of these two theories is true directly from a new experiment.

Consider the case that there is only the $E_i^{(G)}$ component in the gravitational field; then

$$\mathbf{F}_1 = m\mathbf{E}^{(G)} \tag{87}$$

No matter what mass and velocity the particles have, they have the same acceleration. Hence, there is an accelerative frame in which all particles are in inertial motion. This is the case which general relativity discusses. In the case that there is only a $B_z^{(G)}$ component in the gravitational field, the gravitational force of (21) is

$$\mathbf{F}_2 = m\mathbf{u} \times \mathbf{B}^{(G)}/c \tag{88}$$

If two particles with the same mass pass through a gravitational field along the x and y axes, respectively, then the directions of the gravitational force exerted on both particles are along the $-y$ and x axes. It is clear that we cannot find an accelerative frame such that all particles are in inertial motion. If this phenomenon is observed in experiment, then the equivalence principle of general relativity is wrong.

The property of $\mathbf{F}_2$ of (88) is similar to that of the magnetic force. On the other hand, since both the gravitational $\mathbf{B}^{(G)}$ field and the magnetic field are due to a moving field source, these two fields are analogous. The special relativistic gravitational theory predicts that there is only a $\mathbf{B}^{(G)}$ field in the system of Figure 1 with two rings revolving around their common axis. From (59), we have

$$B_z^{(G)} = \frac{-6GM_0\gamma^2 vR}{c(R^2+b^2)^{3/2}} \tag{89}$$

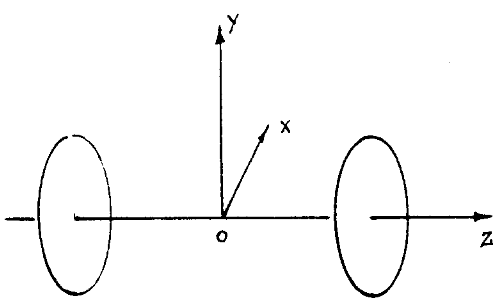

Fig. 1

and all other field strengths are zero at both ring centers, where M_0, R, and V are the mass, radius, and velocity of revolution of the rings, respectively, and $2b$ is the distance between the two ring centers. As mentioned above, $B_z^{(G)} \propto v/c$ and $F_2 \propto (u/c) B_z^{(G)} \cos\alpha$; this is the main reason we have not observed the effect of F_2 in investigations.

While Weinberg also discussed the problem of the tenability of the principle of equivalence, the difference between his discussion and that introduced above should be apparent.

In conclusion, the experimental results are more favorable to the special relativistic gravitational theory than to general relativity. Still, new experiments are needed to test which theory is correct.

REFERENCES

Atwater, H. A. (1974). *Introduction to General Relativity*, Oxford University Press.

Clifford, M. W. (1981). *Theory and Experiment in Gravitational Physics*, Cambridge University Press.

Jackson, J. D. (1976). *Classical Electrodynamics*, New York.

Landau, L. D., and Lifshitz, E. M. (1975). *The Classical Theory of Fields*, Oxford University Press.

Stacey, F. D., *et al.* (1987*a*). *Review of Modern Physics*, **59**, 157.

Stacey, F. D., Tuck, G. J., and Moore, G. I. (1987*b*). *Physical Review*, **36**, 2374(D).

Stephani, H. (1982). *General Relativity*, Cambridge University Press.